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## DECAYS OF EXCITED STRANGE MESONS IN THE EXTENDED NJL MODEL

*M.K.Volkov<sup>1</sup>, V.L.Yudichev<sup>2</sup>, D.Ebert<sup>3</sup>*

A chiral  $U(3) \times U(3)$  Lagrangian, containing besides the usual meson fields their first radial excitations, is considered. The Lagrangian is derived by bosonization of the Nambu–Jona-Lasinio (NJL) quark model with separable nonlocal interactions. The spontaneous breaking of chiral symmetry is governed by the NJL gap equation. The first radial excitations of the kaon,  $K^*$  and  $\phi$  are described with the help of two form factors. The values for the decay widths of the processes  $K^{*'} \rightarrow \rho K$ ,  $K^{*'} \rightarrow K^* \pi$ ,  $K^{*'} \rightarrow K \pi$ ,  $\phi' \rightarrow K^* K$ ,  $\phi' \rightarrow \bar{K} K$ ,  $K' \rightarrow K \rho$ ,  $K' \rightarrow K^* \pi$  and  $K' \rightarrow K 2\pi$  are obtained in qualitative agreement with the experimental data.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

## Распады возбужденных состояний странных мезонов в расширенной модели НИЛ

*М.К.Волков, В.Л.Юдичев, Д.Эберт*

Рассмотрен киральный  $U(3) \times U(3)$ -лагранжиан, содержащий кроме обычных мезонных полей их первые радиальные возбуждения. Лагранжиан получен бозонизацией кварковой модели типа Намбу–Иона-Лазинио (НИЛ) с нелокальным взаимодействием сепарабельного типа. Спонтанное нарушение киральной симметрии обусловлено уравнением на щель. Первые радиальные возбуждения каона,  $K^*$  и  $\phi$  описаны при помощи двух факторов. Величины ширины распадов для процессов  $K^{*'} \rightarrow \rho K$ ,  $K^{*'} \rightarrow K^* \pi$ ,  $K^{*'} \rightarrow K \pi$ ,  $\phi' \rightarrow K^* K$ ,  $\phi' \rightarrow \bar{K} K$ ,  $K' \rightarrow K \rho$ ,  $K' \rightarrow K^* \pi$  и  $K' \rightarrow K 2\pi$  получены в качественном согласии с экспериментальными данными.

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### 1. Introduction

In our previous papers [1,2,3] the chiral quark model of the Nambu–Jona-Lasinio (NJL) type with separable nonlocal interactions has been proposed. This model is a nonlocal extension of the standard NJL model [4,5,6,7,8]. The first radial excitations of the

<sup>1</sup>E-mail: volkov@thsunl.jinr.ru

<sup>2</sup>E-mail: yudichev@thsunl.jinr.ru

<sup>3</sup>Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstrasse 110, D-10115 Berlin, Germany; E-mail: debert@qft2.physik.hu-berlin.de

scalar, pseudoscalar, vector and axial-vector mesons were described with the help of form factors corresponding to 3-dimensional ground and excited state wave functions. The meson masses, weak decay constants and a set of decay widths of nonstrange mesons were calculated.

The theoretical foundations for the choice of polynomial pion-quark form factors were discussed in [1] and it was shown that we can choose these form factors in such a way that the mass gap equation conserves its usual form and gives a solution with a constant constituent quark mass. Moreover, the quark condensate does not change after including the excited states in the model, because the tadpoles connected with the excited scalar fields vanish. Thus, in this approach it is possible to describe radially excited mesons above the usual NJL vacuum, preserving the usual mechanism of chiral symmetry breaking. Finally, it has been shown that one can derive an effective meson Lagrangian for the ground and excited meson states directly in terms of local fields and their derivatives. A nonlocal separable interaction is defined in the Minkowski space in a 3-dimensional (yet covariant) way whereby form factors depend only on the part of the quark-antiquark relative momentum transverse to the meson momentum. This ensures absence of spurious relative-time excitations [9].

In the paper [2], the meson mass spectrum for the ground and excited pions, kaons and the vector meson nonet in the  $U(3) \times U(3)$  model of this type has been obtained. By fitting the meson mass spectrum, all parameters in this model are fixed. This then allows one to describe all the strong, electromagnetic and weak interactions of these mesons without introducing any new additional parameter.

In the paper [3], it was shown that this model satisfactorily describes two types of decays. This concerns strong decays like  $\rho \rightarrow 2\pi$ ,  $\pi' \rightarrow \rho\pi$ ,  $\rho' \rightarrow 2\pi$  associated with divergent quark diagrams as well as the decays  $\rho' \rightarrow \omega\pi$  and  $\omega' \rightarrow \rho\pi$  defined by anomalous quark diagrams. Here we continue the similar calculations for the description of the decay widths of strange pseudoscalar and vector mesons.

The paper is organized as follows. In section 2, we introduce the effective quark interaction in the separable approximation and describe its bosonization. In section 3, we derive the effective Lagrangian for the pions and kaons and perform the diagonalization leading to the physical pion and kaon ground and excited states. In section 4, we carry out the diagonalization for the  $K^*$  and  $\phi$  mesons. In section 5, we give the parameters of our model and the masses of the ground and excited states of kaons,  $K^*$  and  $\phi$  mesons and the weak decay constants  $F_\pi$ ,  $F_{\pi'}$ ,  $F_K$  and  $F_{K^*}$ . In section 6, we evaluate the decay widths of the processes  $K^{*'} \rightarrow K^*\pi$ ,  $K^{*'} \rightarrow \rho K$ ,  $K^{*'} \rightarrow K\pi$ ,  $\phi' \rightarrow K^*K$ , and  $\phi' \rightarrow \bar{K}K$ . In section 7, we calculate the decay widths of the processes  $K' \rightarrow \rho K$ ,  $K' \rightarrow K^*\pi$  and  $K' \rightarrow K2\pi$ . The obtained results are discussed in section 8.

## 2. $U(3) \times U(3)$ Chiral Lagrangian with the Excited Meson States

We shall use a separable interaction, which is still of current-current form, but allows for nonlocal vertices (form factors) in the definition of the quark currents

$$L[\bar{q}, q] = \int d^4x \bar{q}(x)(i\cancel{\partial} - m^0)q(x) + \bar{L}_{\text{int}}, \quad (1)$$

$$\tilde{L}_{\text{int}} = \int d^4x \sum_{a=0}^8 \sum_{i=1}^N \left[ \frac{G_1}{2} [j_{S,i}^a(x) j_{S,i}^a(x) + j_{P,i}^a(x) j_{P,i}^a(x)] - \frac{G_2}{2} [j_{V,i}^{a,\mu}(x) j_{V,i}^{a,\mu}(x) + j_{A,i}^{a,\mu}(x) j_{A,i}^{a,\mu}(x)] \right], \quad (2)$$

$$j_{S,i}^a(x) = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{S,i}^a(x; x_1, x_2) q(x_2), \quad (3)$$

$$j_{P,i}^a(x) = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{P,i}^a(x; x_1, x_2) q(x_2), \quad (4)$$

$$j_{V,i}^{a,\mu}(x) = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{V,i}^{a,\mu}(x; x_1, x_2) q(x_2), \quad (5)$$

$$j_{A,i}^{a,\mu}(x) = \int d^4x_1 \int d^4x_2 \bar{q}(x_1) F_{A,i}^{a,\mu}(x; x_1, x_2) q(x_2). \quad (6)$$

Here  $m^0$  is the current quark mass matrix (we suppose that  $m_u^0 \approx m_d^0$ );  $j_{S,P,V,A}^a(x)$  denote, respectively, the scalar, pseudoscalar, vector and axial-vector currents of the quark field ( $U(3)$ -flavor);  $F_{U,i}^{a,\mu}(x; x_1, x_2)$ ,  $i = 1, \dots, N$ , are a set of nonlocal scalar, pseudoscalar, vector and axial-vector quark vertices (in general momentum- and spin-dependent), which will be specified below.

Upon bosonization we obtain [1,2]

$$\begin{aligned} L_{\text{bos}}(\bar{q}, q; \sigma, \phi, P, A) = & \int d^4x_1 \int d^4x_2 \bar{q}(x_1) \left[ (i\partial\!\!\!/_{x_2} - m^0) \delta(x_1 - x_2) + \right. \\ & + \int d^4x \sum_{a=0}^8 \sum_{i=1}^N (\sigma_i^a(x) F_{\sigma,i}^a(x; x_1, x_2) + \phi_i^a(x) F_{\phi,i}^a(x; x_1, x_2) + \\ & + V_i^{a,\mu}(x) F_{V,i}^{a,\mu}(x; x_1, x_2) + A_i^{a,\mu}(x) F_{A,i}^{a,\mu}(x; x_1, x_2)) \left. \right] q(x_2) - \\ & - \int d^4x \sum_{i=1}^N \left[ \frac{1}{2G_1} (\sigma_i^{a2}(x) + \phi_i^{a2}(x)) - \frac{1}{2G_2} (V_i^{a,\mu2}(x) + A_i^{a,\mu2}(x)) \right]. \end{aligned} \quad (7)$$

This Lagrangian describes a system of local meson fields,  $\sigma_i^a(x)$ ,  $\phi_i^a(x)$ ,  $V_i^{a,\mu}(x)$ ,  $A_i^{a,\mu}(x)$ ,  $i = 1, \dots, N$ , which interact with the quarks through nonlocal vertices. These fields are not yet to be associated with physical particles, which will be obtained after determining the vacuum and diagonalizing the effective meson Lagrangian.

In order to describe the first radial excitations of mesons ( $N=2$ ), we take the form factors in the form (see [1])

$$\begin{aligned} F_{\sigma,2}^a(\mathbf{k}) &= \lambda^a f_a^P(\mathbf{k}), \quad F_{\phi,2}^a(\mathbf{k}) = i\gamma_5 \lambda^a f_a^P(\mathbf{k}), \\ F_{V,2}^{a,\mu}(\mathbf{k}) &= \gamma^\mu \lambda^a f_a^V(\mathbf{k}), \quad F_{A,2}^{a,\mu}(\mathbf{k}) = \gamma_5 \gamma^\mu \lambda^a f_a^V(\mathbf{k}) \end{aligned} \quad (8)$$

$$f_a^U(\mathbf{k}) = c_a^U (1 + d_a \mathbf{k}^2). \quad (9)$$

$\lambda^a$  are the Gell-Mann matrices. We consider here the form factors in the momentum space and in the rest frame of the mesons ( $\mathbf{P}_{\text{meson}} = 0$ ;  $k$  and  $P$  are the relative and total momentum of the quark-antiquark pair). For the ground states of the mesons we choose the functions  $f_a^{U,0}(\mathbf{k}) = 1$ .

After integrating over the quark fields in Eq.(7), one obtains the effective Lagrangian of the  $\sigma_1^a, \sigma_2^a, \phi_1^a, \phi_2^a, V_1^{a,\mu}, V_2^{a,\mu}, A_1^{a,\mu}$  and  $A_2^{a,\mu}$  fields

$$L(\sigma', \phi, V, A, \bar{\sigma}, \bar{\phi}, \bar{V}, \bar{A}) = -\frac{1}{2G_1}(\sigma_a^2 + \phi_a^2 + \bar{\sigma}_a^2 + \bar{\phi}_a^2) + \frac{1}{2G_2}(V_a^2 + A_a^2 + \bar{V}_a^2 + \bar{A}_a^2) - \\ - iN_c \text{Tr} \log [i\cancel{D} - m^0 + (\sigma'_a + i\gamma_5 \phi_a + \gamma_\mu V_a^\mu + \gamma_5 \gamma_\mu A_a^\mu + \\ + (\bar{\sigma}_a + i\gamma_5 \bar{\phi}_a) f_a^P + (\gamma_\mu \bar{V}_a^\mu + \gamma_5 \gamma_\mu \bar{A}_a^\mu) f_a^V) \lambda^a]. \quad (10)$$

Let us define the vacuum expectation of the  $\sigma'_a$  fields\*

$$\langle \frac{\delta L}{\delta \sigma'_a} \rangle_0 = -iN_c \text{tr} \int_{\Lambda_3} \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^0 + \langle \sigma'_a \rangle_0)} - \frac{\langle \sigma'_a \rangle_0}{G_1} = 0. \quad (11)$$

Introduce the new sigma fields whose vacuum expectations are equal to zero

$$\sigma_a = \sigma'_a - \langle \sigma'_a \rangle_0 \quad (12)$$

and redefine the quark masses

$$m_a = m_a^0 - \langle \sigma'_a \rangle_0. \quad (13)$$

Then Eq.(11) can be rewritten in the form of the usual gap equation

$$m_i = m_i^0 + 8G_1 m_i I_1(m_i), \quad (i = u, d, s), \quad (14)$$

where

$$I_n(m_i) = -iN_c \int_{\Lambda_3} \frac{d^4 k}{(2\pi)^4} \frac{1}{(m_i^2 - k^2)^n} \quad (15)$$

and  $m_i$  are the constituent quark masses.

### 3. The Effective Lagrangian for the Ground and Excited States of the Pions and Kaons

To describe the first excited states of all the meson nonets, it is necessary to use three different slope parameters  $d_a$  in the form factors  $f_a^U(\mathbf{k})$  (see Eq.(9))

$$f_{uu}^{P,V}(\mathbf{k}) = c_{uu}^{P,V} (1 + d_{uu} \mathbf{k}^2),$$

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\*We can derive this form of the gap equation only if the condition  $\langle \bar{\sigma}_a \rangle_0 = 0$  is fulfilled (see Refs.1,2,3 and Eqs.(17)).

$$f_{us}^{P,V}(\mathbf{k}) = c_{us}^{P,V}(1 + d_{us} \mathbf{k}^2), \quad (16)$$

$$f_{ss}^{P,V}(\mathbf{k}) = c_{ss}^{P,V}(1 + d_{ss} \mathbf{k}^2),$$

Following our works [1,2] we can fix the parameters  $d_{uu}$ ,  $d_{us}$  and  $d_{ss}$  by using the conditions

$$I_1^{f_{uu}^P}(m_u) = 0, \quad I_1^{f_{us}^P}(m_u) + I_1^{f_{us}^P}(m_s) = 0, \quad I_1^{f_{ss}^P}(m_s) = 0, \quad (17)$$

where

$$I_1^{f_a^P}(m_i) = -iN_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f_a \cdot f_a}{(m_i^2 - k^2)}. \quad (18)$$

Eqs. (17) allow us to conserve the gap equations in the form usual for the NJL model (see Eqs. (14)), because the tadpoles with the excited scalar external fields do not contribute to the quark condensates and to the constituent quark masses.

Using Eqs. (17) we obtain close values for all  $d_a$

$$d_{uu} = -1.784 \text{ GeV}^{-2}, \quad d_{us} = -1.7565 \text{ GeV}^{-2}, \quad d_{ss} = -1.727 \text{ GeV}^{-2}. \quad (19)$$

Now let us consider the free part of the Lagrangian (10). For the pions and kaons we obtain

$$L^{(2)}(\phi) = \frac{1}{2} \sum_{ij=1}^2 \sum_{a=1}^7 \phi_i^a(P) K_{ij}^{ab}(P) \phi_j^b(P). \quad (20)$$

Here

$$\sum_{a=1}^3 (\phi_i^a)^2 = (\pi_i^0)^2 + 2\pi_i^+ \pi_i^-, \quad (\phi_i^4)^2 + (\phi_i^5)^2 = 2K_i^+ K_i^-, \quad (\phi_i^6)^2 + (\phi_i^7)^2 = 2K_i^0 \bar{K}_i^0. \quad (21)$$

The quadratic form  $K_{ij}^{ab}(P)$ , Eq.(20), is obtained as

$$K_{ij}^{ab}(P) \equiv \delta^{ab} K_{ij}^a(P),$$

$$K_{ij}^a(P) = -\delta_{ij} \frac{1}{G_1} - iN_c \text{tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{\not{k} + \frac{1}{2} \not{P} - m_q^a} i\gamma_5 f_i^a \frac{1}{\not{k} - \frac{1}{2} \not{P} - m_{q'}^a} i\gamma_5 f_j^a \right], \quad (22)$$

$$f_1^a \equiv 1, \quad f_2^a \equiv f_a^P(\mathbf{k}).$$

$$m_q^a = m_u \quad (a = 1, \dots, 7); \quad m_{q'}^a = m_u \quad (a = 1, \dots, 3); \quad m_{q'}^a = m_s \quad (a = 4, \dots, 7), \quad (23)$$

$m_u$  and  $m_s$  are the constituent quark masses ( $m_u \approx m_d$ ). The integral (22) is evaluated by expanding in the meson field momentum  $P$ . To order  $P^2$ , one obtains

$$K_{11}^a(P) = Z_1^a(P^2 - M_1^2), \quad K_{22}^a(P) = Z_2^a(P^2 - M_2^2) \quad (24)$$

$$K_{12}^a(P) = K_{21}^a(P) = \gamma^a(P^2 - \Delta^2 \delta_{ab|b=4,\dots,7}), \quad (\Delta = m_s - m_u),$$

where

$$Z_1^a = 4I_2^a Z, \quad Z_2^a = 4I_2^{fa} \bar{Z}, \quad \gamma^a = 4I_2^{fa} Z, \quad (25)$$

$$M_1^{a2} = (Z_1^a)^{-1} \left[ \frac{1}{G_1} - 4(I_1^a(m_q^a) + I_1^a(m_{q'}^a)) \right] + Z^{-1} \Delta^2 \delta_{ab} |_{b=4,\dots,7}, \quad (26)$$

$$M_2^{a2} = (Z_2^a)^{-1} \left[ \frac{1}{G_1} - 4(I_1^{fa}(m_q^a) + I_1^{fa}(m_{q'}^a)) \right] + \bar{Z}^{-1} \Delta^2 \delta_{ab} |_{b=4,\dots,7}, \quad (27)$$

Here,  $Z = 1 - \frac{6m_u^2}{M_{a1}^2} \approx 0.7$ ,  $\bar{Z} = 1 - \Gamma^2 \frac{6m_u^2}{M_{a1}^2} \approx 1$  (see Eq.(32)),  $Z$  is the additional renorma-

lization of the ground pseudoscalar meson states, taking into account the  $\phi^a \rightarrow A^a$  transitions (see [2,3]).  $I_n^a$ ,  $I_n^{fa}$  and  $I_n^{ffa}$  denote the usual loop integrals arising in the momentum expansion of the NJL quark determinant, but now with zero, one or two factors  $f_a(\mathbf{k})$ , Eqs.(16), in the numerator (see (18) and below)

$$I_2^{f \cdot fa}(m_q, m_{q'}) = -iN_c \int_{\Lambda_3} \frac{d^4 k}{(2\pi)^4} \frac{f_a(\mathbf{k}) \dots f_a(\mathbf{k})}{(m_q^2 - k^2)(m_{q'}^2 - k^2)}. \quad (28)$$

After the renormalization of the meson fields

$$\phi^{ar} = \sqrt{Z^a} \phi_i^a \quad (29)$$

the part of the Lagrangian (20), describing the pions and kaons, takes the form

$$L_\pi^{(2)} = \frac{1}{2} \left[ (P^2 - M_\pi^2) \pi_1^2 + 2\Gamma_\pi P^2 \pi_1 \pi_2 + (P^2 - M_\pi^2) \pi_2^2 \right], \quad (30)$$

$$L_K^{(2)} = \frac{1}{2} \left[ (P^2 - M_{K_1}^2 - \Delta^2) K_1^2 + (P^2 - M_{K_2}^2 - \Delta^2) K_2^2 + 2\Gamma_K (P^2 - \Delta^2) K_1 K_2 \right]. \quad (31)$$

Here

$$\Gamma_a = \frac{\gamma_5}{\sqrt{Z_1^a Z_2^a}} = \frac{I_2^{fa} \sqrt{\bar{Z}}}{\sqrt{I_2^a I_2^{fa} \bar{Z}}}. \quad (32)$$

After the transformations of the meson fields

$$\phi^a = \cos(\theta_a - \theta_a^0) \phi_1^{ar} - \cos(\theta_a + \theta_a^0) \phi_2^{ar}, \quad (33)$$

$$\phi^{a'} = \sin(\theta_a - \theta_a^0) \phi_1^{ar} - \sin(\theta_a + \theta_a^0) \phi_2^{ar}$$

the Lagrangians (30) and (31) take the diagonal forms

$$L_\pi^{(2)} = \frac{1}{2} (P^2 - M_\pi^2) \pi^2 + \frac{1}{2} (P^2 - M_\pi^2) \pi'^2, \quad (34)$$

$$L_K^{(2)} = \frac{1}{2} (P^2 - M_K^2) K^2 + \frac{1}{2} (P^2 - M_{K'}^2) K'^2. \quad (35)$$

Here

$$M_{(\pi,\pi)}^2 = \frac{1}{2(1-\Gamma_\pi^2)} \left[ M_{\pi_1}^2 + M_{\pi_2}^2 (-,+) \sqrt{(M_{\pi_1}^2 - M_{\pi_2}^2)^2 + (2M_{\pi_1} M_{\pi_2} \Gamma_\pi)^2} \right], \quad (36)$$

$$M_{(K,K')}^2 = \frac{1}{2(1-\Gamma_K^2)} \left[ M_{K_1}^2 + M_{K_2}^2 + 2\Delta^2(1-\Gamma_K^2) (-,+) \sqrt{(M_{K_1}^2 - M_{K_2}^2)^2 + (2M_{K_1} M_{K_2} \Gamma_K)^2} \right], \quad (37)$$

and

$$\tan 2\bar{\theta}_a = \sqrt{\frac{1}{\Gamma_a^2} - 1} \left[ \frac{M_{\phi_1}^2 - M_{\phi_2}^2}{M_{\phi_1}^2 + M_{\phi_2}^2} \right], \quad 2\theta_a = 2\bar{\theta}_a + \pi, \quad (38)$$

$$\sin \theta_a^0 = \sqrt{\frac{1 + \Gamma_a}{2}} \quad (39)$$

$$M_{\pi_1}^2 = (4ZI_2(m_u, m_u))^{-1} \left[ \frac{1}{G_1} - 8I_1(m_u) \right] = \frac{m_u^0}{4Zm_u I_2(m_u, m_u) G_1}, \quad (40)$$

$$M_{\pi_2}^2 = (4ZI_2^{\text{ff}}(m_u, m_u))^{-1} \left[ \frac{1}{G_1} - 8I_1^{\text{ff}}(m_u) \right],$$

$$\begin{aligned} M_{K_1}^2 &= (4ZI_2(m_u, m_s))^{-1} \left[ \frac{1}{G_1} - 4(I_1(m_u) + I_1(m_s)) \right] + (Z^{-1} - 1)\Delta^2 = \\ &= \frac{\frac{m_u^0}{m_u} + \frac{m_s^0}{m_s}}{4ZI_2(m_u, m_s) G_1} + (Z^{-1} - 1)\Delta^2, \end{aligned} \quad (41)$$

$$M_{K_2}^2 = (4I_2^{\text{ff}}(m_u, m_s))^{-1} \left[ \frac{1}{G_1} - 4(I_1^{\text{ff}}(m_u) + I_1^{\text{ff}}(m_s)) \right].$$

#### 4. The Effective Lagrangian for the Ground and Excited States of the Vector Mesons

The free part of the effective Lagrangian (10) describing the ground and excited states of the vector mesons has the form

$$L^{(2)}(V) = -\frac{1}{2} \sum_{ij=1}^2 \sum_{a=0}^8 V_i^{\mu a}(P) R_{ij}^{\mu\nu a}(P) V_j^{\nu a}(P), \quad (42)$$

where

$$\sum_{a=0}^3 V_i^{\mu a} = (\omega_i^\mu)^2 + (\rho_i^{0\mu})^2 + 2\rho_i^{+\mu}\rho_i^{-\mu}, \quad (V_i^{4\mu})^2 + (V_i^{5\mu})^2 = 2K_i^{*+\mu}K_i^{*-\mu}, \quad (43)$$

$$(V_i^{6\mu})^2 + (V_i^{7\mu})^2 = 2K_i^{*0\mu}K_i^{*0\mu}, \quad (V_i^{8\mu})^2 = (\phi_i^\mu)^2$$

and

$$R_{ij}^{\mu\nu a}(P) = -\frac{\delta_{ij}}{G_2} g^{\mu\nu} - iN_c \text{tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{k + \frac{1}{2}P - m_q^a} \gamma^\mu f_i^{a,V} \frac{1}{k - \frac{1}{2}P - m_q^a} \gamma^\nu f_j^{a,V} \right], \quad (44)$$

$$f_1^{a,V} \equiv 1, \quad f_2^{a,V} \equiv f_a^V(\mathbf{k}).$$

To order  $P^2$ , one obtains

$$R_{11}^{\mu\nu a} = W_1^a [P^2 g^{\mu\nu} - P^\mu P^\nu - g^{\mu\nu} (\overline{M}_1^a)^2],$$

$$R_{22}^{\mu\nu a} = W_2^a [P^2 g^{\mu\nu} - P^\mu P^\nu - g^{\mu\nu} (\overline{M}_2^a)^2], \quad (45)$$

$$R_{12}^{\mu\nu a} = R_{21}^{\mu\nu a} = \overline{\gamma}^a [P^2 g^{\mu\nu} - P^\mu P^\nu - \frac{3}{2} \Delta^2 g^{\mu\nu} \delta^{ab} |_{b=4..7}],$$

Here

$$W_1^a = \frac{8}{3} I_2^a, \quad W_2^a = \frac{8}{3} I_2^{fa}, \quad \overline{\gamma}^a = \frac{8}{3} I_2^{fa}, \quad (46)$$

$$(\overline{M}_1^a)^2 = (W_1^a G_2)^{-1} + \frac{3}{2} \Delta^2 \delta^{ab} |_{b=4..7}, \quad (47)$$

$$(\overline{M}_2^a)^2 = (W_2^a G_2)^{-1} + \frac{3}{2} \Delta^2 \delta^{ab} |_{b=4..7}. \quad (48)$$

After renormalization of the meson fields

$$V_i^{\mu ar} = \sqrt{W_i^a} V_i^{\mu a} \quad (49)$$

we obtain the Lagrangians

$$L_\rho^{(2)} = -\frac{1}{2} [(g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} M_{\rho_1}^2) \rho_1^\mu \rho_1^\nu + 2\Gamma_\rho (g^{\mu\nu} P^2 - P^\mu P^\nu) \rho_1^\mu \rho_2^\nu + (g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} M_{\rho_2}^2) \rho_2^\mu \rho_2^\nu], \quad (50)$$

$$L_\phi^{(2)} = -\frac{1}{2} [(g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} M_\phi^2) \phi_1^\mu \phi_1^\nu + 2\Gamma_\phi (g^{\mu\nu} P^2 - P^\mu P^\nu) \phi_1^\mu \phi_2^\nu + (g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} M_{\phi_2}^2) \phi_2^\mu \phi_2^\nu], \quad (51)$$

$$L_{K^*}^{(2)} = -\frac{1}{2} \left[ (g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} \left( \frac{3}{2} \Delta^2 + M_{K_1^*}^2 \right)) K_1^{*\mu} K_1^{*\nu} + \right.$$



$$\begin{aligned}
& + 2\Gamma_{K^*} \left( g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} \frac{3}{2} \Delta^2 \right) K_1^{*\mu} K_2^{*\nu} + \\
& + \left( g^{\mu\nu} P^2 - P^\mu P^\nu - g^{\mu\nu} \left( \frac{3}{2} \Delta^2 + M_{K_2^*}^2 \right) \right) K_2^{*\mu} K_2^{*\nu} \Big].
\end{aligned} \tag{52}$$

Here

$$\begin{aligned}
M_{\rho_1}^2 &= \frac{3}{8G_2 I_2(m_u, m_u)}, & M_{K_1^*}^2 &= \frac{3}{8G_2 I_2(m_u, m_s)}, \\
M_{\phi_1}^2 &= \frac{3}{8G_2 I_2(m_s, m_s)}, & M_{\rho_2}^2 &= \frac{3}{8G_2 I_2^{\text{ff}}(m_u, m_u)},
\end{aligned} \tag{53}$$

$$\begin{aligned}
M_{K_2^*}^2 &= \frac{3}{8G_2 I_2^{\text{ff}}(m_u, m_s)}, & M_{\phi_2}^2 &= \frac{3}{8G_2 I_2^{\text{ff}}(m_s, m_s)}, \\
\Gamma_{a_{ij}} &= \frac{I_2^a(m_i, m_j)}{\sqrt{I_2^a(m_i, m_i) I_2^a(m_j, m_j)}}.
\end{aligned} \tag{54}$$

After transformations of the vector meson fields, similar to Eqs.(33) for the pseudoscalar mesons, the Lagrangians (50,51,52) take the diagonal form

$$L_{V^a, \bar{V}^a}^{(2)} = -\frac{1}{2} [(g^{\mu\nu} P^2 - P^\mu P^\nu - M_{V^a}^2) V^{a\mu} V^{a\nu} + (g^{\mu\nu} P^2 - P^\mu P^\nu - M_{\bar{V}^a}^2) \bar{V}^{a\mu} \bar{V}^{a\nu}], \tag{55}$$

where  $V^a$  and  $\bar{V}^a$  are the physical ground and excited states of vector mesons

$$M_{\rho, \bar{\rho}}^2 = \frac{1}{2(1 - \Gamma_\rho^2)} \left[ M_{\rho_1}^2 + M_{\rho_2}^2 (-, +) \sqrt{(M_{\rho_1}^2 - M_{\rho_2}^2)^2 + (2M_{\rho_1} M_{\rho_2} \Gamma_\rho)^2} \right], \tag{56}$$

$$M_{\phi, \bar{\phi}}^2 = \frac{1}{2(1 - \Gamma_\phi^2)} \left[ M_{\phi_1}^2 + M_{\phi_2}^2 (-, +) \sqrt{(M_{\phi_1}^2 - M_{\phi_2}^2)^2 + (2M_{\phi_1} M_{\phi_2} \Gamma_\phi)^2} \right], \tag{57}$$

$$\begin{aligned}
M_{K^*, \bar{K}^*}^2 &= \frac{1}{2(1 - \Gamma_{K^*}^2)} \left[ M_{K_1^*}^2 + M_{K_2^*}^2 + 3\Delta^2(1 - \Gamma_{K^*}^2) \right. \\
& \left. (-, +) \sqrt{(M_{K_1^*}^2 - M_{K_2^*}^2)^2 + (2M_{K_1^*} M_{K_2^*} \Gamma_{K^*})^2} \right],
\end{aligned} \tag{58}$$

## 5. Model Parameters and Meson Masses

In the paper [2], there were obtained numerical estimations for the model parameters, meson masses and weak decay constants in our model. Here we only give the values: the constituent quark masses are  $m_u \approx m_d \approx 280$  MeV,  $m_s \approx 455$  MeV; the cut-off parameter  $\Lambda_3 = 1.03$  GeV; the four-quark coupling constants  $G_1 = 3.47$  GeV<sup>-2</sup> and  $G_2 = 12.5$  GeV<sup>-2</sup>;

the slope parameters in the form factors  $d_{uu} = -1.784 \text{ GeV}^{-2}$ ,  $d_{us} = -1.757 \text{ GeV}^{-2}$ ,  $d_{ss} = -1.727 \text{ GeV}^{-2}$ ; the external parameters in the form factors  $c_{uu}^\pi = 1.37$ ,  $c_{uu}^\rho = 1.32$ ,  $c_{us}^K = 1.45$ ,  $c_{us}^{K^*} = 1.54$  and  $c_{ss}^\phi = 1.41$ .

With the model parameters fixed, we obtain the angles  $\theta_a$  and  $\theta_a^0$ :

$$\theta_\pi = 59.48^\circ, \quad \theta_\pi^0 = 59.12^\circ, \quad \theta_\rho = 81.8^\circ, \quad \theta_\rho^0 = 81.5^\circ, \quad \theta_K = 60.2^\circ, \quad (59)$$

$$\theta_K^0 = 57.13^\circ, \quad \theta_{K^*} = 84.7^\circ, \quad \theta_{K^*}^0 = 59.14^\circ, \quad \theta_\phi = 68.4^\circ, \quad \theta_\phi^0 = 57.13^\circ,$$

and the meson masses are

$$M_\pi = 136 \text{ MeV}, \quad M_{\pi'} = 1.3 \text{ GeV}, \quad M_\rho = 768 \text{ MeV}, \quad M_{\rho'} = 1.49 \text{ GeV} \quad (60)$$

$$M_K = 496 \text{ MeV}, \quad M_{K'} = 1.45 \text{ GeV}, \quad M_{K^*} = 887 \text{ MeV}, \quad M_{K^*} = 1.479 \text{ GeV}$$

The experimental values are [10]

$$M_{\pi^\pm}^{\text{exp}} = 134.9764 \pm 0.0006 \text{ MeV}, \quad M_{\pi'}^{\text{exp}} = 1300 \pm 100 \text{ MeV}, \quad M_\rho^{\text{exp}} = 768.5 \pm 0.6 \text{ MeV}, \\ M_{\rho'}^{\text{exp}} = 1465 \pm 25 \text{ MeV}, \quad M_{K^\pm}^{\text{exp}} = 493.677 \pm 0.016 \text{ MeV}, \quad M_{K'}^{\text{exp}} \approx 1460 \text{ MeV}, \quad (61)$$

$$M_{K^*}^{\text{exp}} = 891.59 \pm 0.24 \text{ MeV}, \quad M_{K^*}^{\text{exp}} = 1412 \pm 12 \text{ MeV}.$$

For the weak decay constants we have

$$F_\pi = 93 \text{ MeV}, \quad F_{\pi'} = 0.57 \text{ MeV}, \quad F_K = 108 \text{ MeV} \approx 1.16 F_\pi, \quad F_{K'} = 3.3 \text{ MeV}, \quad (62)$$

Now we can calculate the strong decay widths of  $K'$  and  $K^*$

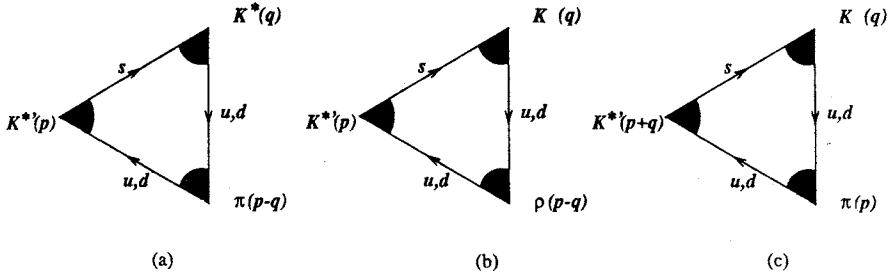
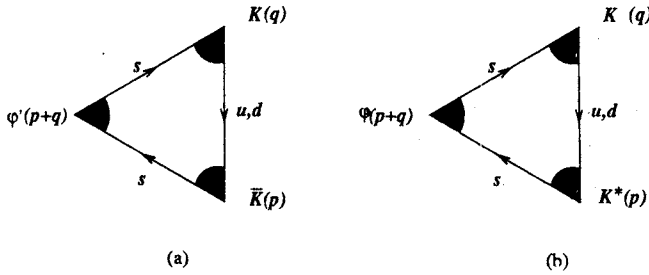
## 6. The Decays $K^{*'} \rightarrow K\rho$ , $K^{*'} \rightarrow K^*\pi$ , $K^{*'} \rightarrow K\pi$ , $\phi' \rightarrow K^*K$ , $\phi' \rightarrow \bar{K}K$

In the framework of our model, the decay modes of the excited strange vector mesons  $K^{*'}$  and  $\phi'$  are represented by the triangular diagrams shown in Figs.1 and 2. When calculating these diagrams, we keep the least possible dependence on the external momenta: squared for the anomaly type graphs and linear for another type. We omit here the higher order momentum dependence.

As it has been mentioned in this paper, every vertex is now momentum dependent and includes form factors defined in Sec.3 (see Eq.(16)). In Figs.1 and 2 the presence of form factors is marked by black shaded angles in vertices. Each black shaded vertex with a pseudoscalar meson is implied to contain the following linear combination for the ground state:

$$f_a = \frac{1}{\sin 2\theta_a^0} \left[ \frac{\sin(\theta_a + \theta_a^0)}{\sqrt{Z_1^a}} + \frac{\sin(\theta_a - \theta_a^0)}{\sqrt{Z_2^a}} f_a \right], \quad (63)$$

and for an excited state —


 Fig.1. The one-loop diagrams set for the decays of  $K^*$ 

 Fig.2. The one-loop diagrams set for the decays of  $\phi'$ 

$$\bar{f}'_a = \frac{-1}{\cos 2\theta_a^0} \left[ \frac{\sin(\theta_a + \theta_a^0)}{\sqrt{Z_1^a}} + \frac{\cos(\theta_a - \theta_a^0)}{\sqrt{Z_2^a}} f_a \right], \quad (64)$$

$\theta_a$  and  $\theta_a^0$  are the angles defined in Sec.3 (see Eqs.(38) and (39)) and  $f_a$  is one of the form factors defined in Sec.3 (Eq.(16)). In the case of vector meson vertices, we have the same linear combinations except that  $Z_i^a$  are to be replaced by  $W_i^a$  (46), and the related angles and form factor parameters must be chosen.

Now we can calculate the decay widths of the excited mesons. Let us start with the process  $K^{*'} \rightarrow K^* \pi$ . The corresponding amplitude,  $T_{K^{*'} \rightarrow K^* \pi}$ , has the form

$$T_{K^{*'} \rightarrow K^* \pi} = g_{K^{*'} \rightarrow K^* \pi} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \xi^\mu(p|\lambda) \xi^\nu(q|\lambda'), \quad (65)$$

where  $p$  and  $q$  are the momenta of the  $K^{*'}$  and  $K^*$  mesons, respectively, and  $g_{K^{*'} \rightarrow K^* \pi}$  is the (dimensional) coupling constant, which follows from the combination of one-loop integrals

$$g_{K^{*'} \rightarrow K^* \pi} = \frac{8m_s}{m_u^2 - m_s^2} \left( I_2^{\bar{f}'_K \bar{f}_K \bar{f}_\pi}(m_u) - I_2^{\bar{f}'_{K^*} \bar{f}_{K^*} \bar{f}_\pi}(m_u, m_s) \right). \quad (66)$$

Note that in (66) the integrals  $I_2^{f..f}$  are defined in the same way as in Sec.3, Eq.(28), except that the form factors  $f$  in (28) are replaced by the expressions of type (63) and (64).  $\bar{\xi}_\mu(p|\lambda)$  and  $\bar{\xi}_\nu(q|\lambda')$  are polarized vector wave functions ( $\lambda, \lambda' = 1, 2, 3$ ).

Using (63) and (64) we expand the above expression and rewrite it in terms of  $I_2^{ff}$  defined in (28). The result is too lengthy, so we omit it here. For the parameters given in Sec.5 we find

$$g_{K^{*'} \rightarrow K^* \pi} = 3.6 \text{ GeV}^{-1} \quad (67)$$

and the decay width

$$\Gamma_{K^{*'} \rightarrow K^* \pi} = \frac{g_{K^{*'} \rightarrow K^* \pi}^2}{4\pi} |\mathbf{p}|^3 = 70 \text{ MeV}. \quad (68)$$

Here  $|\mathbf{p}|$  is the 3-momentum of a produced particle in the rest frame of the decaying meson. The lower limit for this value, coming from experiment, is  $\sim 91 \pm 9 \text{ MeV}$  [10].

Similar calculation is to be performed for the rest of the  $K^{*'}$  decay modes under consideration. The coupling constant  $g_{K^{*'} \rightarrow K\rho}$  derived in the same way as in (66), with the only difference that  $\bar{f}_\pi$  and  $\bar{f}_{K^*}$  are to be replaced by  $\bar{f}_\rho$  and  $\bar{f}_K$ . The corresponding amplitude,  $T_{K^{*'} \rightarrow K\rho}$ , takes the form

$$T_{K^{*'} \rightarrow K\rho} = g_{K^{*'} \rightarrow K\rho} \varepsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \bar{\xi}^\mu(p|\lambda) \bar{\xi}^\nu(p-q|\lambda'), \quad (69)$$

where  $p$  and  $q$  are the momenta of  $K^{*'}$  and  $K$  mesons, respectively, and

$$g_{K^{*'} \rightarrow K\rho} = \frac{8m_s}{m_u^2 - m_s^2} \left( I_2^{\bar{f}'_K \bar{f}_K \bar{f}_\rho}(m_u) - I_2^{\bar{f}'_K \bar{f}_K \bar{f}_\rho}(m_u, m_s) \right). \quad (70)$$

The corresponding decay width follows from (69) and (70) via integration of the squared module of the decay amplitude over the phase space of the final state

$$\Gamma_{K^{*'} \rightarrow K\rho} = \frac{g_{K^{*'} \rightarrow K\rho}^2}{4\pi} |\mathbf{p}|^3. \quad (71)$$

For the parameters given in Sec. 5 one has

$$g_{K^{*'} \rightarrow K\rho} = 3.2 \text{ GeV}^{-1}, \quad (72)$$

$$\Gamma_{K^{*'} \rightarrow K\rho} = 23 \text{ MeV}. \quad (73)$$

From experiment, the upper limit for this process is  $\Gamma_{K^{*'} \rightarrow K\rho}^{\text{exp}} < 16 \pm 1.5 \text{ MeV}$ .

The process  $K^{*'} \rightarrow K\pi$  is described by the amplitude

$$T_{K^{*'} \rightarrow K\pi} = \frac{ig_{K^{*'} \rightarrow K\pi}}{2} (p-q)^\mu \bar{\xi}_\mu(p+q|\lambda), \quad (74)$$

where  $p$  and  $q$  are the momenta of  $K_\pi$  and  $K$ , and  $\bar{\xi}_\mu$  is the  $K^{*'}$  wave function. The coupling constant  $g_{K^{*'} \rightarrow K\pi}$  is obtained by calculating the one-loop integral

$$g_{K^{*'} \rightarrow K\pi} = 4I_2^{\bar{f}'\kappa^* \bar{f}_K \bar{f}_\pi}(m_u, m_s) = 2.3 \quad (75)$$

and the decay width is

$$\Gamma_{K^{*'} \rightarrow K\pi} = \frac{g_{K^{*'} \rightarrow K\pi}^2 |\mathbf{p}|^3}{8\pi M_{K^{*'}}^2} = 24 \text{ MeV}. \quad (76)$$

The experimental value is  $15 \pm 5$  MeV [10].

The mesons with hidden strangeness ( $\phi'$ ) are treated in the same way as  $K^{*'}$ . We consider the two decay modes:  $\phi' \rightarrow KK^*$  and  $\phi' \rightarrow \bar{K}\bar{K}$ . Their amplitudes are

$$T_{\phi' \rightarrow KK^*} = g_{\phi' \rightarrow KK^*} \varepsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \xi^\mu(p+q|\lambda) \xi^\nu(p|\lambda') \quad (77)$$

$$T_{\phi' \rightarrow \bar{K}\bar{K}} = ig_{\phi' \rightarrow \bar{K}\bar{K}} (p-q)_\mu \xi^\mu(p+q|\lambda). \quad (78)$$

Here  $\xi_\mu$  and  $\xi_\nu$  are the wave functions of the  $\phi'$  and  $K^*$  mesons,  $p$  and  $q$  are the momenta of the  $K$  and  $K^*$  mesons. The related coupling constants are

$$g_{\phi' \rightarrow KK^*} = \frac{8m_u}{m_s^2 - m_u^2} \left( I_2^{\bar{f}'\phi \bar{f}_K^* \bar{f}_K}(m_s) - I_2^{\bar{f}'\phi \bar{f}_K^* \bar{f}_K}(m_u, m_s) \right), \quad (79)$$

$$g_{\phi' \rightarrow \bar{K}\bar{K}} = 4I_2^{\bar{f}'\phi \bar{f}_K \bar{f}_K}(m_s). \quad (80)$$

Thus, the decay widths are estimated as

$$\Gamma_{\phi' \rightarrow KK^*} = 91 \text{ MeV}, \quad (81)$$

$$\Gamma_{\phi' \rightarrow \bar{K}\bar{K}} = 11 \text{ MeV}. \quad (82)$$

Unfortunately, there are no reliable data from experiment on the partial decay widths for  $\phi' \rightarrow KK^*$  and  $\phi' \rightarrow \bar{K}\bar{K}$  except the total width of  $\phi'$  which is estimated as  $150 \pm 50$  MeV [10]. However, the dominance of the process  $\phi' \rightarrow KK^*$  is observed, which is in agreement with our result.

## 7. The decays $K' \rightarrow K^*\pi$ , $K' \rightarrow K\rho$ , $K' \rightarrow K\pi\pi$

Following the scheme outlined in the previous section, we first estimate the  $K' \rightarrow K^*\pi$  and  $K' \rightarrow K\rho$  decay widths (Fig.3). Their amplitudes are

$$T_{K' \rightarrow K^*\pi} = ig_{K' \rightarrow K^*\pi} (p+q)^\mu \xi_\mu(p-q|\lambda), \quad (83)$$

$$T_{K' \rightarrow K\rho} = ig_{K' \rightarrow K\rho} (p+q)^\mu \xi_\mu(p-q|\lambda), \quad (84)$$

here  $p$  is the momentum of  $K'$ ,  $q$  is the momentum of  $\pi(K)$ , and  $\xi_\mu$  is the vector meson wave function. The coupling constants are

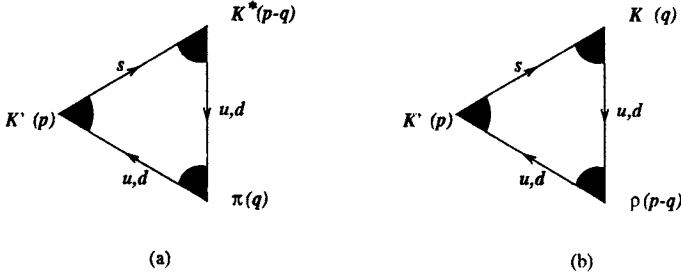


Fig.3. The one-loop diagrams set for the decays of  $K'$

$$g_{K' \rightarrow K^* \pi} = 4I_2^{\bar{f}' \bar{f} \bar{f}} \kappa^{\bar{f} \bar{f} \bar{f}} \pi(m_u, m_s), \quad (85)$$

$$g_{K' \rightarrow K \rho} = 4I_2^{\bar{f}' \bar{f} \bar{f}} \kappa^{\bar{f} \bar{f} \bar{f}} \rho(m_u, m_s), \quad (86)$$

By calculating the integrals in the above formulae we have  $g_{K' \rightarrow K^* \pi} = -1.3$  and  $g_{K' \rightarrow K \rho} = -1.18$ . The decay widths thereby are

$$\Gamma_{K' \rightarrow K^* \pi} = 90 \text{ MeV}, \quad (87)$$

$$\Gamma_{K' \rightarrow K \rho} = 51 \text{ MeV}. \quad (88)$$

These processes have been seen in experiment and the decay widths are [10]

$$\Gamma_{K' \rightarrow K^* \pi}^{\text{exp}} \sim 109 \text{ MeV}, \quad (89)$$

$$\Gamma_{K' \rightarrow K \rho}^{\text{exp}} \sim 34 \text{ MeV}. \quad (90)$$

The remaining decay  $K' \rightarrow K\pi\pi$  into three particles requires more complicated calculations. In this case, one must consider a box diagram Fig.4. (a) and two types of diagrams Fig.4.(b,c) with intermediate  $\sigma$  and  $K_0^*$  resonances. The diagrams for resonant

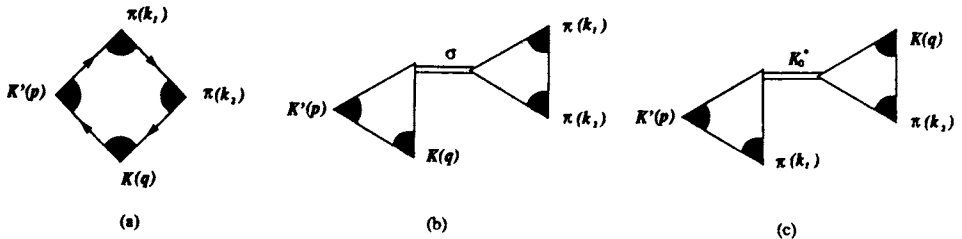


Fig.4. The one-loop diagrams set for the decays of  $K' \rightarrow K2\pi$

channels are approximated with the use of the relativistic Breit–Wigner function. The integration over the kinematically relevant range in the phase space for final states gives

$$\Gamma_{K' \rightarrow K\pi\pi} \sim 1 \text{ MeV.} \quad (91)$$

## 8. Summary and Conclusions

In the standard NJL model for the description of interaction of mesons, it is conventional to use the one-loop quark approximation, where the external momentum dependence in quark loops is neglected. This allows one to obtain, in this approximation, the chiral symmetric phenomenological Lagrangian [4,5,6,7,8], which gives a good description of the low-energy meson physics in the energy range of an order of 1 GeV [11]. In this paper, we have used a similar method for the description of interaction of the excited mesons. In so far as the masses of the excited mesons noticeably exceed 1 GeV, we pretend only to a qualitative description rather than quantitative agreement with the experiment. For the light excited mesons, we have got the results closer to the experiment [3], while for heavier strange mesons we are only in qualitative agreement with experimental data. One should note that the description of all the decays has been obtained without introducing new parameters, besides those used for the description of the mass spectrum.

In this work, we have shown that the dominant decays of the excited vector mesons  $K^{* \prime}$  and  $\phi'$  are the decays  $K^{* \prime} \rightarrow K^* \pi$  ( $\rho K$ ) and  $\phi' \rightarrow KK^*$ , which go through the triangle quark loops of the anomaly type. These results are in qualitative agreement with experimental data [10]. The decays of the type  $K^{* \prime} \rightarrow K\pi$  and  $\phi' \rightarrow K\bar{K}$ , going through the other (not anomaly type) quark diagrams, have smaller decay widths, which is also in agreement with experiment.

On the other hand, the main decays of the  $K'$  meson  $K' \rightarrow K^* \pi$ ,  $K' \rightarrow K\rho$  and  $K' \rightarrow K\pi\pi$  are described by the quark diagrams, which are similar to those for the decay  $\pi' \rightarrow \rho\pi$  (see [3]). The dominant decays here are the decays  $K' \rightarrow K^* \pi$  and  $K' \rightarrow K\rho$ . These results are also in qualitative agreement with the recent experimental data. So one can see that our model satisfactorily describes not only the masses and weak decay constants of the radially excited mesons [1, 2] but also their decay widths.

We would like to emphasize once more that we did not use any additional parameter for description of the decays [3]. The model is too simple to pretend to a more exact quantitative description of the meson decay widths.

A similar calculation has also been made in the  $3P_1$  potential model [12]. Nonlocal versions of chiral quark models for the description of excited meson states have been also considered in various works (see, for instance [13,14]).

In our further work we are going to describe the masses and decay widths of the excited states of  $\eta$  and  $\eta'$  mesons.

## Acknowledgment

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